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14. ABSTRACT
We have introduced a modification of the classical Euler numerical scheme for stochastic differential equations driven by a fractional Brownian motion with Hurst parameter larger than 1/2. For this new scheme, we have derived a precise rate of convergence to zero or the error and the limit in distribution of the error fluctuations. We have studied time discrete numerical schemes based on Taylor expansions for rough differential equations and for stochastic differential equations driven by a fractional Brownian motion with Hurst parameter larger than 1/2.

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Report Title

Final Report: Stochastic Evolution Equations Driven by Fractional Noises

ABSTRACT

We have introduced a modification of the classical Euler numerical scheme for stochastic differential equations driven by a fractional Brownian motion with Hurst parameter larger than 1/2. For this new scheme, we have derived a precise rate of convergence to zero or the error and the limit in distribution of the error fluctuations. We have studied time discrete numerical schemes based on Taylor expansions for rough differential equations and for stochastic differential equations driven by a fractional Brownian motion with Hurst parameter larger than 1/2.

We have studied the linear stochastic heat equation driven by a general multiplicative Gaussian noise. This equation is a continuous version of the parabolic Anderson model, which is a popular model for diffusions with a random potential, with many applications in mathematical physics. The existence and uniqueness of a solution have been established under general conditions on covariance of the noise. On the other hand, Feynman-Kac formulas for the solutions and for their moments have been derived and applied to obtain sharp intermittency upper and lower bounds for the moments of the solution. We also have derived upper and lower bounds for intermittency fronts. We have established the existence and uniqueness of a solution for the non-linear one-dimensional stochastic heat equation driven by a Gaussian noise which is white in time and it has the covariance of a fractional Brownian motion with Hurst parameter between 1/4 and 1/2 in the space variable. The roughness of this noise creates important technical difficulties that we have been able to solve. For this equation we have derived precise spatial asymptotic results and large deviation estimates.

We have established central limit theorems for functionals of fractional Volterra processes, which are relevant in the estimation of parameters for the fractional CARk model, using the fourth moment theorem. We have proved a central limit theorem for functionals of a large class of Gaussian self-similar processes and we have established a decomposition result for self-similar Gaussian processes arising form stochastic partial differential equations with additive noise. Finally we have derived a general Ito formula in law for weak symmetric integrals with respect to the fractional Brownian motion, we have studied stochastic differential equations with power type nonlinearities and we have established asymptotic properties of the derivative of the self-intersection local time of the fractional Brownian motion.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

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(b) Papers published in non-peer-reviewed journals (N/A for none)

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(c) Presentations

"Stochastic heat equation with rough multiplicative noise"
Presented at the Conference in honor of Professor Vlad Bally: Stochastic Calculus, Monte Carlo Methods and Mathematical Finance, University of Le Mans, October 6-9, 2015.

"Parabolic Anderson model driven by colored noise"
Presented at the Workshop on deterministic and stochastic partial differential equations, Brown University, November 6-8, 2015.

"Parabolic Anderson model driven by colored noise"
Presented at the 2015 CMS Winter Meeting, Montreal, Canada, December 4-7, 2015.

"Stochastic heat equation driven by a rough time-fractional noise"
Presented at the AMS Sectional Meeting, University of Utah, April 9-10, 2016.

"Stochastic heat equation with rough multiplicative noise"
Presented at the Swiss Probability Seminar, June 6-7, 2016.

"Approximation schemes for stochastic differential equations driven by a fractional Brownian motion"
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Received		Paper
11/22/2016	1.00	Yaozhong Hu, Yanghui Liu, David Nualart. Rate of convergence and asymptotic error distribution in Euler approximation schemes for fractional diffusions, Annals of Applied Probability (06 2013)
11/22/2016	7.00	Daniel Harnett, David Nualart. Decomposition and limit theorems for a class of self-similar Gaussian processes, Progress in Probability (08 2015)
11/22/2016	2.00	Yaozhong Hu, Jingyu Huang, David Nualart, Samy Tindel. Stochastic heat equations with general multiplicative Gaussian noises: Holder continuity and intermittency, Electronic Journal of Probability (02 2014)
11/22/2016	3.00	Yaozhong Hu, Jingyu Huang, Khoa Le, David Nualart, Samy Tindel. Stochastic heat equation with rough dependence in space, Annals of Probability (05 2015)
11/22/2016	4.00	Yaozhong Hu, Jingyu Huang, David Nualart. On the intermittency front of stochastic heat equation driven by colored noises, Electronic Communications in Probability (06 2015)
11/22/2016	5.00	Ivan Nourdin, David Nualart, Rola Zintout. Multivariate central limit theorems for averages of fractional Volterra processes and applications to parameter estimation, Statistical Inference for Stochastic Processes (02 2015)
11/23/2016	6.00	David Nualart, Daniel Harnett. Central limit theorem for functionals of a generalized self-similar Gaussian process, Stochastic Processes and Their Applications (08 2015)

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Scientific Progress

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Scientific Progress and Accomplishments

Research proposal 1. Approximation schemes for stochastic differential equations driven by a fractional Brownian motion

Foreword

Consider a d-dimensional stochastic differential equation driven by an m-dimensional fractional Brownian motion $B^H = \{B_t^H, t \in [0,T]\}$ with Hurst parameter $H \in \left(\frac{1}{2}, 1\right)$ of the form

$$X_t = x_0 + \int_0^t b(X_s)ds + \sum_{j=1}^m \int_0^t \sigma^j(X_s)dB_s^j. \tag{1}$$

The stochastic integrals appearing in the above formula are path-wise Riemann-Stieltjes integrals constructed using Young's methodology (see [31]). We have studied numerical approximations for the solution to equation (1) based on uniform partitions of the interval $[0,T]$, $t_i = \frac{iT}{n}$, $i = 0, \dots, n$. It was proved by Mishura in 2008 (see [26]) that the classical Euler approximation, denoted by X^n, satisfies

$$\sup_{0 \le t \le T} |X_t^n - X_t| \le C_\epsilon n^{1-2H+\epsilon},$$

for any real number $\epsilon > 0$. Moreover, the convergence rate n^{1-2H} is sharp for this scheme, in the sense that $n^{2H-1}[X_t^n - X_t]$ converges almost surely to a finite and non-zero limit. This shows that the numerical scheme X^n has a rate of convergence different from the Euler-Maruyama scheme for the classical Brownian motion, which is not surprising because for $H = \frac{1}{2}$ the sequence X_t^n converges to the solution of the corresponding Itô equation, and for $H > \frac{1}{2}$ we are dealing with Riemann-Stieltjes pathwise integrals.

We have introduced a new (modified Euler) approximation scheme for equation (1) that takes into account the correlation between the input noise dB_t^H/dt and the system X_t. This new scheme has the rate of convergence γ_n^{-1}, where

$$\gamma_n = \begin{cases} n^{2H-\frac{1}{2}} & \text{if} \quad \frac{1}{2} < H < \frac{3}{4}, \\ \frac{n}{\sqrt{\log n}} & \text{if} \quad H = \frac{3}{4}, \\ n & \text{if} \quad \frac{3}{4} < H < 1\,. \end{cases} \tag{2}$$

In particular, the rate of convergence becomes $n^{-\frac{1}{2}}$ when H is formally set to $\frac{1}{2}$, which matches the rate of convergence of the Euler scheme in the case of the classical Brownian motion. Furthermore, we have derived the asymptotic behavior of the fluctuations of the error. The proof of these results is based on the techniques of Malliavin calculus or stochastic calculus of variations.

In the second part of this project, we have studied two variations of the time discrete Taylor schemes for rough differential equations and for stochastic differential equations driven by fractional Brownian motions.

Results

The results obtained in this research project are included in following papers:

[1] Y. Hu, Y. Liu and D. Nualart: Rate of convergence and asymptotic error distribution in Euler approximation schemes for fractional diffusions, *Annals of Applied Probability* **26**, no. 2, 1147-1207, 2016.

Here is a summary of the main results proved in this paper. First we have established the following result on the rate of convergence for the modified Euler approximation scheme, that we denote by X^n:

Theorem 1 *Let X^n be the modified Euler approximation scheme for Equation (1). We assume that $b : \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma : \mathbb{R}^d \to \mathbb{R}^{d\times d}$ are three and four times continuously differentiable, respectively, with bounded partial derivatives. Then for any $p \geq 1$ there exists a constant C independent of n (but dependent on p) such that*

$$\sup_{0\leq t\leq T} \mathbb{E}\left[|X_t^n - X_t|^p\right]^{\frac{1}{p}} \leq C\gamma_n^{-1}.$$

In the case $H \in (\frac{1}{2}, \frac{3}{4}]$ we have derived a central theorem for the fluctuations of the error. More precisely, the process $\gamma_n(X - X^n)$ converges in law to the solution of a linear stochastic differential equation driven by a matrix-valued Brownian motion independent of B as n tends to infinity. A formal statement is included in the next theorem.

Theorem 2 *Let $H \in (\frac{1}{2}, \frac{3}{4}]$ and let X^n be the modified Euler approximation scheme for Equation (1). Assume that b and σ are four and five times continuously differentiable, respectively, with bounded partial derivatives. Then the following convergence in law the space $C([0,T])$ holds as n tends to infinity,*

$$\{\gamma_n(X_t - X_t^n)\,, t \in [0,T]\} \to \{U_t\,, t \in [0,T]\}\,, \tag{3}$$

where $\{U_t\,, t \in [0,T]\}$ is the solution of the linear d-dimensional stochastic differential equation

$$U_t = \int_0^t \nabla b(X_s) U_s ds + \sum_{j=1}^m \int_0^t \nabla \sigma^j(X_s) U_s dB_s^j + \sum_{i,j=1}^m \int_0^t (\nabla \sigma^j \sigma^i)(X_s) dW_s^{ij}\,.$$

In this equation W is a matrix-valued Brownian motion independent of the fractional Brownian motion B^H.

The proof of this theorem is based on a limit theorem for weighted sums and a central limit theorem for quadratic functionals of the fractional Brownian motion, which is established applying the fourth moment theorem of Nualart and Peccati [29].

When we let the Hurst parameter H converge to $\frac{1}{2}$ we obtain formally the classical results for the Brownian motion, obtained, for instance by Kurtz and Protter in [22].

In case $H \in (\frac{3}{4}, 1)$, using a limit theorem in L^p for weighted sums, we have derived the L^p-limit of the normalized error $n(X_t - X_t^n)$ in the case $H \in (\frac{3}{4}, 1)$.

Theorem 3 *Let $H \in (\frac{3}{4}, 1)$ and σ and b are four and five times continuously differentiable, respectively, with bounded partial derivatives. Then*

$$n(X_t - X_t^n) \to \bar{U}_t$$

in $L^p(\Omega)$ as n tends to infinity, where $\{\bar{U}_t\,, t \in [0,T]\}$ is the solution of the following linear stochastic differential equation

$$\begin{aligned}\bar{U}_t &= \int_0^t \nabla b(X_s) \bar{U}_s ds + \sum_{j=1}^m \int_0^t \nabla \sigma^j(X_s) \bar{U}_s dB_s^j + \sum_{i,j=1}^m \int_0^t (\nabla \sigma^j \sigma^i)(X_s) dZ_s^{ij} \\ &\quad + \frac{T}{2} \int_0^t (\nabla b b)(X_s) ds + \frac{T}{2} \int_0^t (\nabla b \sigma)(X_s) dB_s + \frac{T}{2} \sum_{j=1}^m \int_0^t (\nabla \sigma^j b)(X_s) dB_s^j,\end{aligned}$$

where Z is a matrix-valued generalized Rosenblatt-type process.

We have also derived the weak rate of convergence for the modified Euler scheme X^n to equation (1).

Theorem 4 *Let X^n be the modified Euler approximation scheme for equation (1). We assume $b : \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma : \mathbb{R}^d \to \mathbb{R}^{d \times d}$ are three and four times continuously differentiable, respectively, with bounded partial derivatives. Then for any function $f : \mathbb{R}^d \to \mathbb{R}$,* *which is three times continuously differentiable with bounded derivatives, there exists a constant C independent of n such that*

$$\sup_{0\le t\le T}\Big|\mathbb{E}\big[f(X_t)\big]-\mathbb{E}\big[f(X_t^n)\big]\Big|\le Cn^{-1}. \tag{4}$$

If we assume that b and σ are four and five times continuously differentiable, respectively, with bounded partial derivatives, then for each $t\in[0,T]$,

$$\begin{aligned}
&n\Big\{\mathbb{E}\big[f(X_t)\big]-\mathbb{E}\big[f(X_t^n)\big]\Big\} \quad \textit{converges to}\\
&\frac{\alpha_H^2 T}{2}\sum_{j,i=1}^m\int_0^t\int_0^t\int_0^t \mathbb{E}\Big\{D_u^iD_r^j\Big[\nabla f(X_t)\Lambda_t\Gamma_s\big(\nabla\sigma^j\sigma^i\big)(X_s)\Big]\Big\}|u-s|^{2H-2}|s-r|^{2H-2}dudsdr\\
&+\frac{T}{2}\mathbb{E}\Big\{\nabla f(X_t)\Lambda_t\Big[\int_0^t\Gamma_s(\nabla bb)(X_s)ds+\int_0^t\Gamma_s(\nabla b\sigma)(X_s)dB_s\\
&\qquad\qquad+\sum_{j=1}^m\int_0^t\Gamma_s(\nabla\sigma^jb)(X_s)dB_s^j\Big]\Big\}, \qquad \textit{as} \quad n\to\infty.
\end{aligned}$$

[**2**] Y. Hu, Y. Liu and D. Nualart: Taylor schemes for rough differential equations and fractional diffusions, *Discrete and Continuous Dynamical Systems Series B* **21**, no. 9, 3115-3162, 2016.

In this paper, we study two variations of the time discrete Taylor schemes for rough differential equations and for stochastic differential equations driven by fractional Brownian motions. One is the incomplete Taylor scheme which excludes some terms of an Taylor scheme in its recursive computation so as to reduce the computation time. The other one is to add some deterministic terms to an incomplete Taylor scheme to improve the mean rate of convergence. Almost sure rate of convergence and L^p-rate of convergence are obtained for the incomplete Taylor schemes. Almost sure rate is expressed in terms of the Hölder exponents of the driving signals and the L^p-rate is expressed by the Hurst parameters. Both rates involves with the incomplete Taylor scheme in a very explicit way and then provide us with the best incomplete schemes, depending on that one needs the almost sure convergence or one needs L^p-convergence. As in the smooth case, general Taylor schemes are always complicated to deal with. The incomplete Taylor scheme is even more sophisticated to analyze. A new feature of our approach is the explicit expression of the error functions which will be easier to study. Estimates for multiple integrals and formulas for the iterated vector fields are obtained to analyze the error functions and then to obtain the rates of convergence.

Research proposal 2. Stochastic partial differential equations driven by fractional noises

Foreword

We have studied the linear stochastic heat equation (also called parabolic Anderson model) on $\mathbb{R}^d$ driven by a general multiplicative centered Gaussian noise. This equation can be written as

$$\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u\dot{W}, \quad t > 0, \quad x \in \mathbb{R}^d, \tag{5}$$

with initial condition $u_0(x)$, where u_0 is a continuous and bounded function. In the above equation, $\dot{W}$ is a formal Gaussian field that has a covariance of the form

$$\mathbb{E}\left[\dot{W}_{t,x}\dot{W}_{s,y}\right] = \gamma(s-t)\,\Lambda(x-y), \tag{6}$$

where γ and Λ are general nonnegative and nonnegative definite (generalized) functions satisfying some integrability conditions. The product appearing in the above equation (5) can be interpreted as an ordinary product of the solution $u_{t,x}$ times the noise $\dot{W}_{t,x}$ (which is a distribution). In this case the evolution form of the equation will involve a Stratonovich integral (or path-wise Young integral). The product can also be interpreted as a Wick product and in this case the solution satisfies an evolution equation formulated using the Skorohod integral. We have considered both formulations.

There has been a widespread interest in the model (5) in the recent past, with several motivations for its study. First, it appears naturally in homogenization problems for partial differential equations driven by highly oscillating stationary random fields (see [2, 19, 21]. On the other hand, equation (5) is also related to the KPZ growth model through the Cole-Hopf's transformation. In this context, definitions of the equation by means of renormalization and rough paths techniques have been recently investigated in [15, 18]. There is also a strong connexion between equation (5) and the partition function of directed and undirected continuum polymers. This link has been exploited in [23, 30] and is particularly present in [1], where basic properties of an equation of type (5) are translated into corresponding properties of the polymer. Finally, the multiplicative stochastic heat equation exhibits concentration properties of its energy. This interesting phenomenon is referred to as *intermittency* for the process u solution to (5) (see e.g [12, 13, 14]), and as a *localization* property for the polymer measure [8]. The intermittency property for our model is one of the main result we have obtained.

In this project we have derived existence-uniqueness results, Feynman-Kac representations, chaos expansions and intermittency results for a very wide class of Gaussian noises $\dot{W}$ (including in particular those considered in [4, 10]), for both Skorohod and Stratonovich type equations (5). In particular we have obtained some lower and upper bounds for the moment of oder k, for all $k \geq 2$, which are sharp in the sense that they have the same exponential order as the upper bounds.

A second part of the project has been devoted to study the one-dimensional stochastic partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\kappa}{2}\frac{\partial^2 u}{\partial x^2} + \sigma(u)\dot{W}\,, \quad t \geq 0, \quad x \in \mathbb{R}\,, \tag{7}$$

where by

$$\mathbb{E}[\dot{W}(s,x)\dot{W}(t,y)] = H(2H-1)\delta_0(t-s)|x-y|^{2H-2} \tag{8}$$

with $\frac{1}{4} < H < \frac{1}{2}$. That is, W is a standard Brownian motion in time and a fractional Brownian motion with Hurst parameter H in the space variable. The spatial covariance is not a locally integrable function and the above expression is formal. A rigorous treatment of this covariance requires the introduction of the spectral measure $\mu(d\xi) = c_{1,H}|\xi|^{1-2H}d\xi$, where $c_{1,H}$ is a constant depending on H. The standard methodology (see, for instance, [10]) to handle homogeneous spatial covariances cannot be applied here. In a recent paper, Balan, Jolis and Quer-Sardanyons [6] proved the existence of a unique mild solution for equation (7) in the case $\sigma(u) = au + b$, but their method cannot be extended to general nonlinear coefficients.

We have been able to establish the uniqueness of solutions for a nonlinear coefficient σ, using a truncation argument inspired by the work of Gyöngy and Nualart in [17] on the stochastic Burgers equation on the whole real line driven by a space-time white noise. For the existence, we have applied the methodology developed in the work of Gyöngy in [16] on semi-linear stochastic partial differential equations, which consists in taking approximations obtained by regularizing the noise and using a compactness argument on a suitable space of trajectories, together with the strong uniqueness result. We have also established the Hölder continuity of the solution u in both space and time variables and derived upper bounds for the moments of the solution using a sharp Burkholder's inequality, as well as the matching lower bounds for the second moment by means of a Sobolev embedding argument.

A third part of the project has been devoted to study the position of the high peaks that are farthest away from the origin. The propagation of the farthest high peaks was first considered by Conus and Khoshnevisan in [5] for a one-dimensional heat equation driven by space-time white noise, where it is shown that there are intermittency fronts that move linearly with time as αt. Namely, for any fixed $p \in [2,\infty)$, if α is sufficiently small, then the quantity $\sup_{|x|>\alpha t} \mathbb{E}(|u(t,x)|^p)$ grows exponentially fast as t tends to ∞; whereas the preceding quantity vanishes exponentially fast if α is sufficiently large. To be more precise, the authors of [5] define for every $\alpha > 0$,

$$\mathscr{S}(\alpha) := \limsup_{t\to\infty} \frac{1}{t} \sup_{|x|>\alpha t} \log \mathbb{R}(|u(t,x)|^p)\,, \tag{9}$$

and think of α_L as an intermittency lower front if $\mathscr{S}(\alpha) < 0$ for all $\alpha > \alpha_L$, and of α_U as an intermittency upper front if $\mathscr{S}(\alpha) > 0$ whenever $\alpha < \alpha_U$. In [5] it is shown that for each real number $p \geq 2$, $0 < \alpha_U \leq \alpha_L < \infty$, and when $p = 2$, some bounds for α_L and α_U are given. In a later work by Chen and Dalang [3], it is proved that when $p = 2$, there exists a critical number $\alpha^* = \frac{\lambda^2}{2}$ such that $\mathscr{S}(\alpha) < 0$ when $\alpha > \alpha^*$ while $\mathscr{S}(\alpha) > 0$ when $\alpha < \alpha^*$ (this property was first conjectured in [5]). On the other hand, using a variational approach we have been able to compute the exponential growth indices for any value $p \geq 2$.

Inspired by the aforementioned works, we have studied the multidimensional stochastic heat equation (5) driven by a colored noise, both in space and time, when the solution is interpreted in the Skorohod sense. Due to the presence of the time covariance, the propagation speed of the farthest high peaks may not be linear. Thus, in contrast to (9), the inequality $|x| > \alpha t$ there needs to be replaced by $|x| > \alpha t \theta_t$ for some suitable function θ_t. When Λ is the Riesz kernel, we obtain a better estimate of the intermittency lower front. We also have provided explicit formulas for $\mathscr{S}(\alpha)$, for arbitraty $p \geq 2$, using a variational approach.

Further contributions to stochastic partial differential equations include an explicit formula for the two-point correlation function for the solutions to the stochastic heat equation driven by a space-time white noise, the analysis of fractional in time equations and the spatial asymptotics for the stochastic heat equation driven by a Gaussian noise which is whit in time and it has the covariance of a fractional Brownian motion with Hurst parameter $H \in (\frac{1}{4}, \frac{1}{2})$.

Results

The results obtained in this research project are included in the following papers whose main results are described below.

[**3**] J. Huang, Y. Hu, D. Nualart and S. Tindel: Stochastic heat equations with general multiplicative Gaussian noises: Hölder continuity and intermittency. *Electronic Journal of Probability* **20** (2015) 1-50.

Here is a summary of the main results proved in this paper:

(i) In the Skorohod case, the mild solution to equation (5) has a formal Wiener chaos expansion, which converges in $L^2(\Omega)$, provided γ is locally integrable and the spectral measure μ of the spatial covariance satisfies the following integrability condition (known as Dalang's condition):

$$\int_{\mathbb{R}^d} \frac{\mu(d\xi)}{1+|\xi|^2} < \infty. \tag{10}$$

Moreover, the solution is unique. The proof of this result is based on Fourier analysis techniques, and covers the particular examples of the Riesz kernel and the Bessel kernel considered by Balan and Tudor in [7]. Our results also encompass the case of the fractional covariance $\Lambda(x) = \prod_{i=1}^d H_i(2H_i-1)|x_i|^{2H_i-2}$, where $H_i > \frac{1}{2}$ and condition (10) is satisfied if and only if $\sum_{i=1}^d H_i > d-1$. This particular structure has been examined in [20].

(ii) Under these general hypotheses to ensure the existence and uniqueness of the solution of Skorohod type one cannot expect to have a Feynman-Kac formula for the solution, but one can establish Feynman-Kac-type formulas for the moments of the solution. More precisely, for any integer $k \geq 2$

$$\mathbb{E}\left[u_{t,x}^k\right] = \mathbb{E}_B\left[\prod_{i=1}^k u_0(B_t^i + x)\exp\left(\sum_{1\le i<j\le k}\int_0^t\int_0^t \gamma(s-r)\Lambda(B_s^i - B_r^j)dsdr\right)\right], \tag{11}$$

where $\{B^j; j = 1, \ldots, k\}$ is a family of d-dimensional independent standard Brownian motions independent of W. The formulas we have obtained, generalize those obtained for the Riesz or the Bessel kernels in [7, 20].

(iii) Consider the following more restrictive integrability assumptions on γ and μ: There exists a constant $0 < \beta < 1$ such that for any $t \in \mathbb{R}$,

$$0 \leq \gamma(t) \leq C_\beta |t|^{-\beta}$$

and the measure μ satisfies

$$\int_{\mathbb{R}^d} \frac{\mu(d\xi)}{1+|\xi|^{2-2\beta}} < \infty.$$

Under these assumptions, we have derived a Feynman-Kac formula for the solution u to Equation (5) in the Stratonovich sense. An immediate application of the Feynman-Kac formula is the Hölder continuity of the solution.

(iv) In the Stratonovich case, we have given a notion of solution to equation (5) using two different methodologies. One is based on the Stratonovich integral defined as the limit in probability of the integrals with respect to a regularization of the noise, and another one uses a path-wise approach, weighted Besov spaces and a Young integral approach. We show that the two notions coincide and some existence-uniqueness results which are (to the best of our knowledge) the first link between pathwise and Malliavin calculus solutions to equation (5).

(v) Under some further restrictions, we have obtained some sharp lower bounds for the moments of the solution. Namely, we can find explicit numbers κ_1 and κ_2 and constants c_j, C_j for $j = 1, 2$ such that

$$C_1 \exp\left(c_1 t^{\kappa_1} k^{\kappa_2}\right) \le \mathbb{E}\left[|u_{t,x}|^k\right] \le C_2 \exp\left(c_2 t^{\kappa_1} k^{\kappa_2}\right)$$

for all $t \ge 0$, $x \in \mathbb{R}^d$ and $k \ge 2$.

[4] Y. Hu, J. Huang, K. Lê, D. Nualart and S. Tindel: Stochastic heat equation with rough dependence in space. Under revision for the *Annals of Probability*. Arxiv:1505.04924v1. 57 pages long.

Here is a summary of the main results proved in this paper:

(i) *Uniqueness result for equation (7):* Consider the space $\mathcal{Z}_T^p$ formed by all random fields $v(t,x)$ such that

$$\|v\|_{\mathcal{Z}_T^p} := \sup_{t\in[0,T]} \|v(t,\cdot)\|_{L^p(\Omega\times\mathbb{R})} + \sup_{t\in[0,T]} \mathcal{N}^*_{\frac{1}{2}-H,p} v(t) < \infty, \tag{12}$$

where $p \ge 2$ and

$$\mathcal{N}^*_{\frac{1}{2}-H,p} v(t) = \left(\int_{\mathbb{R}} \|v(t,\cdot) - v(t,\cdot+h)\|^2_{L^p(\Omega\times\mathbb{R})} |h|^{2H-2} dh\right)^{\frac{1}{2}}. \tag{13}$$

Assume that σ is differentiable, its derivative is Lipschitz, $\sigma(0) = 0$ and for $p > \frac{6}{4H-1}$, the initial condition u_0 is in $L^p(\mathbb{R})$ and

$$\int_{\mathbb{R}} \|u_0(\cdot) - u_0(\cdot+h)\|^2_{L^p(\mathbb{R})} |h|^{2H-2} dh < \infty\,. \tag{14}$$

Then, if u and v are two solutions of (7) and $u, v \in \mathcal{Z}_T^p$, for every $t \in [0,T]$ and $x \in \mathbb{R}$, $u(t,x) = v(t,x), a.s.$

(ii) *Existence result for equation (7):* Define

$$\mathcal{N}_\beta u_0(x) = \left(\int_{\mathbb{R}} |u_0(x+h) - u_0(x)|^2 |h|^{-1-2\beta} dh\right)^{\frac{1}{2}}$$

and assume that for some $\beta > \frac{1}{2} - H$ and some $p > \max(\frac{6}{4H-1}, \frac{1}{\beta+H-1/2})$, the initial condition u_0 is in $L^p(\mathbb{R}) \cap L^\infty(\mathbb{R})$ and

$$\sup_{x\in\mathbb{R}} \mathcal{N}_\beta u_0(x) + \left(\int_{\mathbb{R}} \|u_0(\cdot) - u_0(\cdot + h)\|^2_{L^p(\mathbb{R})} |h|^{2H-2} dh\right)^{\frac{1}{2}} < \infty \,. \tag{15}$$

Suppose also that σ is differentiable and the derivative of σ is Lipschitz and $\sigma(0) = 0$. Then there exists a solution u to (7) in the space $\mathcal{Z}^p_T \cap \mathcal{X}^{\frac{1}{2}-H,p}_T$, where $\mathcal{X}^{\frac{1}{2}-H,p}_T$ is the family of random fields u satisfying

$$\sup_{t\in[0,T],\, x\in\mathbb{R}} \|u(t,x)\|_{L^p(\Omega)} + \sup_{t\in[0,T], x\in\mathbb{R}} \left(\int_{\mathbb{R}} \|u(t,x+y) - u(t,x)\|^2_{L^p(\Omega)} |y|^{2H-2} dy\right)^{\frac{1}{2}} < \infty \,.$$

(ii) *Moment estimates for equation (7):* Let $\frac{1}{4} < H < \frac{1}{2}$, and consider the solution u to equation (7) with $\sigma(u) = u$ wit initial condition is $u_0(x) \equiv 1$. Let $n \geq 2$ be an integer, $x \in \mathbb{R}$ and $t \geq 0$. Then there exist some positive constants c_1, c_2, c_3 independent of n, t and κ with $0 < c_1 < c_2$ satisfying

$$\exp(c_1 n^{1+\frac{1}{H}} \kappa^{1-\frac{1}{H}} t) \leq \mathbb{E}[u^n(t,x)] \leq c_3 \exp\left(c_2 n^{1+\frac{1}{H}} \kappa^{1-\frac{1}{H}} t\right) .$$

[**5**] Y. Hu, J. Huang and D. Nualart: On the intermittency front of stochastic heat equation driven by colored noises. *Electronic Communications in Probability* **21**, no 21, 1-13, 2016.

Here is a summary of the main results proved in this paper. Suppose that the spectral measure of the spatial covariance μ satisfies Dalang's condition. For any real number $N > 0$, we define

$$C_N = \int_{|\xi|>N} \frac{\mu(d\xi)}{|\xi|^2}, \quad \text{and} \quad D_N = \mu\left\{\xi : |\xi| \leq N\right\} . \tag{16}$$

On the other hand, we assume that γ is locally integrable, we set $\int_0^t \gamma(s)ds = \Gamma_t$. The next theorem provides an upper bound for the upper intermittency front:

Theorem 5 *Let $u(t,x)$ be the solution to equation (5) driven by a noise W with covariance structure (6). Assume that u_0 is non-negative and supported in the ball $B_M = \{x \in \mathbb{R}^d : |x| \leq M\}$. Set $\theta_t = \sqrt{D_{N_t} C^{-1}_{N_t}}$, where*

$$N_t = \inf\left\{N \geq 0 : C_N \leq \frac{(2\pi)^d}{32(p-1)\lambda^2 \Gamma_t}\right\} . \tag{17}$$

Then, for any integer $p \geq 2$, we have

$$\bar{\nu}(p) := \inf\left\{\varrho > 0 : \limsup_{t\to\infty} \frac{1}{t\theta_t^2} \sup_{|x|\geq \varrho t\theta_t} \log \mathbb{E}[u^p(t,x)] < 0\right\} \leq 1\,. \tag{18}$$

We have derived also a lower bound for the lower intermittency front, but only in the particular case where the spatial covariance is the Riesz kernel.

[**6**] J. Huang, K. Lê and D. Nualart: Large time asymptotics for the parabolic Anderson model driven by spatially correlated noise. To appear in *Annals of the Institut Henri Poincaré.* Arxiv:1509.00897v3 39 pages long.

In this paper we study the linear stochastic heat equation, also known as parabolic Anderson model, in multidimension driven by a Gaussian noise which is white in time and it has a correlated spatial covariance. Examples of such covariance include the Riesz kernel in any dimension and the covariance of the fractional Brownian motion with Hurst parameter $H \in (\frac{1}{4}, \frac{1}{2}]$ in dimension one. First we establish the existence of a unique mild solution and we derive a Feynman-Kac formula for its moments using a family of independent Brownian bridges and assuming a general integrability condition on the initial data. In the second part of the paper we compute Lyapunov exponents, lower and upper exponential growth indices in terms of a variational quantity. The last part of the paper is devoted to study the phase transition property of the Anderson model.

[**7**] Y. Huang, K. Lê and D. Nualart: Large time asymptotics for the parabolic Anderson model driven by space and time correlated noise. Submitted for publication Arxiv:16-7.00682v1 23 pages long.

In this paper we study the linear stochastic heat equation on $\mathbb{R}^\ell$, driven by a Gaussian noise which is colored in time and space. The spatial covariance satisfies general assumptions and includes examples such as the Riesz kernel in any dimension and the covariance of the fractional Brownian motion with Hurst parameter $H \in (\frac{1}{4}, \frac{1}{2}]$ in dimension one. First we establish the existence of a unique mild solution and we derive a Feynman-Kac formula for its moments using a family of independent Brownian bridges and assuming a general integrability condition on the initial data. In the second part of the paper we compute Lyapunov exponents and lower and upper exponential growth indices in terms of a variational quantity.

[**8**] X. Chen, Y. Hu, S. Tindel and D. Nualart: Spatial asymptotics for the parabolic Anderson model driven by a Gaussian rough noise. Submitted for publication. Arxiv:1607.04092v1. 41 pages long.

The aim of this paper is to establish the almost sure asymptotic behavior as the space variable becomes large, for the solution to the one spatial dimensional stochastic heat equation driven by a Gaussian noise which is white in time and which has the covariance structure of a fractional Brownian motion with Hurst parameter $H \in \left(\frac{1}{4}, \frac{1}{2}\right)$ in the space variable.

[**9**] L. Chen, Y. Hu, K. Kalbasi and D. Nualart: Intermittency for the stochastic heat equation driven by fractional noise in time with $H \in (0, 1/2)$. Submitted for publication. Arxiv:1602.05617v1. 22 pages long.

This paper studies the one-dimensional stochastic heat equation driven by a Gaussian noise which is, with respect to time, a fractional Brownian motion with Hurst parameter $H \in (0, 1/2)$. We establish the Feynman-Kac representation of the solution and obtain both lower and upper bounds for the $L^p(\Omega)$ moments.

[**10**] L. Chen, Y. Hu and D. Nualart: Nonlinear stochastic time-fractional slow and fast diffusion equations on $\mathbb{R}^d$. Submitted for publication. Arxiv:1509.07763v1. 43 pages long.

This paper studies the nonlinear stochastic partial differential equation of fractional orders both in space and time variables:

$$\left(\partial^\beta + \frac{\nu}{2}(-\Delta)^{\alpha/2}\right) u(t,x) = I_t^\gamma \left[\rho(u(t,x))\dot{W}(t,x)\right], \quad t > 0,\ x \in \mathbb{R}^d,$$

where $\dot{W}$ is the space-time white noise, $\alpha \in (0, 2]$, $\beta \in (0, 2)$, $\gamma \geq 0$ and $\nu > 0$. Fundamental solutions and their properties, in particular the nonnegativity, are derived and proved. The existence and uniqueness of solution together with the moment bounds of the solution are obtained under Dalang's condition: $d < 2\alpha + \frac{\alpha}{\beta} \min(2\gamma - 1, 0)$. In some cases, the initial data can be measures. When $\beta \in (0, 1]$, we prove the sample path regularity of the solution.

[**11**] L. Chen, Y. Hu and D. Nualart: Two-point correlation function and Feynman-Kac formula for the stochastic heat equation. To appear in *Potential Analysis*. Arxiv:1607.00682v1 23 pages long.

In this paper, we obtain an explicit formula for the two-point correlation function for the solutions to the stochastic heat equation on $\mathbb{R}$. The bounds for p-th moments proved by Chen and Dalang in [9] are simplified. We validate the Feynman-Kac formula for the p-point correlation function of the solutions to this equation with measure-valued initial data, using techniques from Malliavin calculus.

Research proposal 3. Estimation of parameters for stochastic differential equations driven by a fractional Brownian motion

Foreword

We have studied fractional Volterra processes $X_i = \{X_i(t), t \geq 0\}$, $i = 1, \dots, k$, of the form

$$X_i(t) = \int_0^t x_i(t-s) dB^H(s), \quad t \geq 0, \tag{19}$$

where B^H is a fractional Brownian motion with Hurst parameter $H > \frac{1}{2}$, and $x_i : [0, \infty) \to \mathbb{R}$ are measurable functions satisfying suitable integrability conditions.

The special case of $k = 1$ and $x_1(u) = \sigma\, e^{-\theta u}$, with $\sigma, \theta > 0$, corresponds to the fractional Ornstein-Uhlenbeck process. More generally, our model includes the solution to k-th order stochastic differential equation driven by B^H, known as fractional CARk model, which generalizes the model introduced in [24]:

$$\begin{cases} X^{(k)}(t) & = \sum_{j=0}^{k-1} \theta_j X^{(j)}(t) + \sigma \dot{B}^H(t), \quad t > 0 \\ X(0) & = \ldots = X^{(k-1)}(0) = 0 \end{cases}, \tag{20}$$

where the superscript ${}^{(j)}$ denotes j-fold differentiation with respect to t.

Motivated by the parameter estimation in the model (20), we have derived general central limit theorems for functionals of the process (19). The results are applied to the problem of parameter estimation in the fractional CARk model, and the limit matrix covariance is computed explicitly in the particular case $k = 2$ and under some restrictions on the parameters θ_0 and θ_1.

In a second part of this research direction, we have derived a central limit theorem for functionals of a large class of Gaussian self-similar processes. We have also studied self-similar Gaussian process that arise form stochastic partial differential equations with additive noise. For these processes we have established a decomposition in law and a central limit theorem for the Hermite variations of the increments. Finally we have derived a general Itô formula in law for weak symmetric integrals with respect to the fractional Brownian motion, we have studied stochastic differential equations with power type nonlinearities and we have derived properties of the derivative of the self-intersection local time of the fractional Brownian motion.

Results

The results obtained in this research project are included in the following papers whose main results are described below.

[12] I. Nourdin, D. Nualart and R. Zintout: Multivariate central limit theorems for averages of fractional Volterra processes and applications to parameter estimation. Accepted in *Statistical Inference for Stochastic Processes* **19** no. 2, 219-234, 2016.

Here is a summary of the main results proved in this paper: Consider the random vector $U_T = (U_{1,T}, \ldots, U_{k,T})$, where

$$U_{i,T} = \frac{1}{\sqrt{T}} \int_0^T f_i \left(\frac{X_i(t)}{\sigma_i(t)} \right) dt. \tag{21}$$

In this definition, X_i. $i = 1, \ldots, k$, are the fractional Volterra processes defined by (19), f_i, $i = 1, \ldots, k$, are real mesurable functions satisfying

$$\int_{\mathbb{R}} f_i(x) e^{-x^2/2} dx = 0 \quad \text{and} \quad \int_{\mathbb{R}} f_i^2(x) e^{-x^2/2} dx < \infty, \tag{22}$$

and $\sigma_i(t) = \sqrt{E[X_i(t)^2]}$. The second condition in (22) ensures that f_i can be expanded in Hermite polynomials, namely

$$f_i = \sum_{l=0}^{\infty} a_{i,l} H_l \quad \text{with } \textstyle\sum_{l=0}^{\infty} l! a_{i,l}^2 < \infty, \tag{23}$$

whereas from the first one we deduce that $a_{i,0} = 0$.

Then, following the approach developed in Nourdin, Peccati and Podolskij [28] (see also [27, Chapter 7]) we have proved the following result.

Theorem 6 *Let q_i denote the Hermite rank of f_i, that is, the smallest value of l such that the coefficient $a_{i,l}$ of H_l in (23) is different from zero. Set $q_* = \min_{1 \le i \le k} q_i$ and assume that $q_* \ge 2$. Consider $U_T = (U_{1,T}, \ldots, U_{k,T})$, where $U_{i,T}$ is given by (21). If $H \in (\frac{1}{2}, 1 - \frac{1}{2q_*})$ and if the functions x_i defining X_i satisfy both*

$$\int_{\mathbb{R}} \left(\int_{[0,\infty)^2} |x_i(u) x_j(v)| \, |v - u - a|^{2H-2} du dv \right)^{q_i \vee q_j} da < \infty \tag{24}$$

and

$$\eta_i := \sqrt{H(2H-1) \int_{[0,\infty)^2} x_i(u) x_i(v) |v-u|^{2H-2} du dv} \in (0, \infty), \tag{25}$$

for all $i, j = 1, \ldots, k$, then

$$U_T \stackrel{\text{law}}{\to} N_k(0, \Lambda) \qquad \text{as } T \to \infty, \tag{26}$$

where $\Lambda = (\Lambda_{ij})_{1\le i,j\le k}$ *is given by*

$$\Lambda_{ij} = \sum_{l=q_i \vee q_j}^{\infty} a_{i,l} a_{j,l} l! \frac{H^l (2H-1)^l}{\eta_i^l \eta_j^l} \times \int_{\mathbb{R}} \left(\int_{[0,\infty)^2} x_i(u) x_j(v) |v-u-a|^{2H-2} du dv \right)^l da. \tag{27}$$

In Theorem 6 we must divide by a quantity depending on t in (21), namely $\sigma_i(t)$, which is not very convenient for applications. In this sense we have considered the random vector $V_T = (V_{1,T}, \dots, V_{k,T})$ given by

$$V_{i,T} = \frac{1}{\sqrt{T}} \int_0^T f_i \left(\frac{X_i(t)}{\eta_i} \right) dt, \tag{28}$$

where η_i is given in (25), and we have proved the following theorem.

Theorem 7 *Suppose that* $f_i = P_i$, $i = 1, \dots, k$, *are real polynomials and denote by* q_i *the Hermite rank of* P_i. *Set* $q_* = \min_{1\le i\le k} q_i$ *and assume that* $q_* \ge 2$. *Consider* $V_T = (V_{1,T}, \dots, V_{k,T})$ *given by (28). If* $H \in (\frac{1}{2}, 1 - \frac{1}{2q_*})$ *and if the functions* x_i *defining* X_i *satisfy (24), (25) as well as*

$$\int_{[0,\infty)^2} |x_i(u) x_i(v)| \left((u \wedge v) \vee 1 \right) |v-u|^{2H-2} du dv < \infty, \tag{29}$$

then

$$V_T \overset{\text{law}}{\to} N_k(0, \Lambda) \quad \textit{as } T \to \infty, \tag{30}$$

with Λ *still given by (27).*

[13] D. Harnett and D. Nualart: Central limit theorem for functionals of a generalized self-similar Gaussian process. Arxiv:1508.02756v1. Submitted for publication. 16 pages long.

Here is a summary of the main results proved in this paper. Let $X = \{X_t, t \ge 0\}$ denote a centered self-similar Gaussian process with self-similarity parameter $\beta \in (0,1)$. Consider the following conditions on the function ϕ defined $\phi(x) = \mathbb{E}[X_1 X_x]$ for $x \ge 1$, where $\alpha \in (0, 2\beta]$:
(H.1) ϕ has the form $\phi(x) = -\lambda(x-1)^\alpha + \psi(x)$, where $\lambda > 0$, $\psi(x)$ is twice-differentiable on an open set containing $[1,\infty)$, and there is a constant $C \ge 0$ such that for any $x \in (1,\infty)$

1. $|\psi'(x)| \le Cx^{\alpha-1}$;
2. $|\psi''(x)| \le Cx^{-1}(x-1)^{\alpha-1}$; and
3. $\psi(1) = \beta\psi'(1)$, when $\alpha \ge 1$.

(H.2) There are constants $C > 0$ and $1 < \nu \le 2$ such that for all $x \ge 2$,

1. $|\phi'(x)| \le \begin{cases} C(x-1)^{-\nu} & \text{if } \alpha < 1 \\ C(x-1)^{\alpha-2} & \text{if } \alpha \ge 1, \end{cases}$
2. $|\phi''(x)| \le \begin{cases} C(x-1)^{-\nu-1} & \text{if } \alpha < 1 \\ C(x-1)^{\alpha-3} & \text{if } \alpha \ge 1. \end{cases}$

Under these conditions, and using the Fourth Moment Theorem we proved the following result.

Theorem 8 *Suppose a self-similar Gaussian process X which satisfies (H.1) and (H.2) above. For $T > 0$ and integers $n \ge 2$, consider the sequence*

$$F_n = \frac{1}{\sqrt{n}} \sum_{j=0}^{\lfloor nt \rfloor - 1} f(Y_{j,n}),$$

where $f \in L^2(\mathbb{R}, \gamma)$ has Hermite rank $d \ge 2$ and $\gamma = N(0,1)$ and $Y_{j,n} = \frac{\Delta Xj}{\|\Delta Xj\|_{L^2(\Omega)}}$, $\Delta X_j = X_{(j+1)/n} - X_{j/n}$. Then, if $\alpha < 2 - \frac{1}{d}$, the sequence $\{F_n, n \ge 1\}$ converges in distribution to a Gaussian random variable, with mean zero and variance given by $\sigma^2 = \sum_{q=d}^{\infty} c_q^2 \sigma_q^2$, where

$$\sigma_q^2 = 2^{-q} q! T \sum_{m \in \mathbb{Z}} \left(|m+1|^\alpha + |m-1|^\alpha - 2|m|^\alpha \right)^q .$$

[**14**] D. Harnett and D. Nualart: Decomposition and limit theorems for a class of self-similar Gaussian processes. Arxiv:1508.06641v1. Submitted for publication. 16 pages long.

Here is a summary of the main results proved in this paper: Consider a centered Gaussian process $\{X_t, t \ge 0\}$ with covariance

$$R(s,t) = \mathbb{E}[X_s X_t] = \mathbb{E}\left[\left(\int_0^t Z_{t-r} dB_r^H \right) \left(\int_0^s Z_{s-r} dB_r^H \right) \right], \tag{31}$$

where

(i) $B^H = \{B_t^H, t \geq 0\}$ is a fractional Brownian motion with Hurst parameter $H \in (0,1)$.

(ii) $Z = \{Z_t, t > 0\}$ is a zero-mean Gaussian process, independent of B^H, with covariance

$$\mathbb{E}[Z_s Z_t] = (s+t)^{-\gamma}, \tag{32}$$

where $0 < \gamma < 2H$.

In other words, X is a Gaussian process with the same covariance as the process $\{\int_0^t Z_{t-r} dB_r^H, t \geq 0\}$, which is not Gaussian. Our first result is the following decomposition in law of the process X as the sum of a fractional Brownian motion with Hurst parameter $\frac{\alpha}{2} = H - \frac{\gamma}{2}$ plus a process with regular trajectories.

Theorem 9 *The process X has the same law as $\{\sqrt{\kappa} B_t^{\frac{\alpha}{2}} + Y_t, t \geq 0\}$, where*

$$\kappa = \frac{1}{\Gamma(\gamma)} \int_0^\infty \frac{z^{\gamma-1}}{1+z^2}\, dz, \tag{33}$$

$B^{\frac{\alpha}{2}}$ is a fractional Brownian motion with Hurst parameter $\alpha/2$, and Y (up to a constant) is the process introduced by Lei and Nualart in [25], that is, Y is a centered Gaussian process with covariance given by

$$\mathbb{E}\left[Y_t Y_s\right] = \lambda_1 \int_0^\infty y^{-\alpha-1}(1-e^{-yt})(1-e^{-ys})\, dy,$$

where

$$\lambda_1 = \frac{4\pi}{\Gamma(\gamma)\Gamma(2H+1)\sin(\pi H)} \int_0^\infty \frac{\eta^{1-2H}}{1+\eta^2} d\eta.$$

Theorem 10 *Let $q \geq 2$ be an integer and fix a real $T > 0$. Suppose that $\alpha < 2 - \frac{1}{q}$. For $t \in [0,T]$, define,*

$$F_n(t) = n^{-\frac{1}{2}} \sum_{j=0}^{\lfloor nt \rfloor - 1} H_q\left(\frac{\Delta X j}{\|\Delta X j\|_{L^2(\Omega)}}\right),$$

where $H_q(x)$ denotes the qth Hermite polynomial and $\Delta X_j = X_{(j+1)/n} - X_{j/n}$. Then as $n \to \infty$, the stochastic process $\{F_n(t), t \in [0,T]\}$ converges in law in the Skorohod space $D([0,T])$, to a scaled Brownian motion $\{\sigma B_t, t \in [0,T]\}$, where $\{B_t, t \in [0,T]\}$ is a standard Brownian motion and $\sigma = \sqrt{\sigma^2}$ is given by

$$\sigma^2 = \frac{q!}{2^q} \sum_{m \in \mathbb{Z}} \left(|m+1|^\alpha - 2|m|^\alpha + |m-1|^\alpha\right)^q.$$

[15] G. Binotto, I. Nourdin and D. Nualart: Weak symmetric integrals with respect to the fractional Brownian motion. Arxiv:1606.04046v1. Submitted for publication. 21 pages long.

The aim of this paper is to establish the weak convergence, in the topology of the Skorohod space, of the ν-symmetric Riemann sums for functionals of the fractional Brownian motion when the Hurst parameter takes the critical value $H = (4\ell + 2)^{-1}$, where $\ell = \ell(\nu) \geq 1$ is the largest natural number satisfying $\int_0^1 \alpha^{2j} \nu(d\alpha) = \frac{1}{2j+1}$ for all $j = 0, \ldots, \ell - 1$. As a consequence, we derive a change-of-variable formula in distribution, where the correction term is a stochastic integral with respect to a Brownian motion that is independent of the fractional Brownian motion.

[16] J. A. León, D. Nualart and S. Tindel: Young differential equations with power type nonlinearities. Arxiv:1606.02258v1. Submitted for publication. 25 pages long.

In this paper we give several methods to construct nontrivial solutions to the equation $dy_t = \sigma(y_t)\, dx_t$, where x is a γ-Hölder $\mathbb{R}^d$-valued signal with $\gamma \in (1/2, 1)$ and σ is a function behaving like a power function $|\xi|^\kappa$, with $\kappa \in (0, 1)$. In this situation, classical Young integration techniques allow to get existence and uniqueness results whenever $\gamma(\kappa+1) > 1$, while we focus on cases where $\gamma(\kappa + 1) \leq 1$. Our analysis then relies on Zähle's extension of Young's integral allowing to cover the situation at hand.

[17] A. Jaramillo and D. Nualart: Asymptotic properties of the derivative of self-intersection local time of fractional Brownian motion. To appear in *Stochastic Processes and Their Applications*. Arxiv:1512.07219v1. 34 pages long.

Let $\{B_t\}_{t\geq 0}$ be a fractional Brownian motion with Hurst parameter $\frac{2}{3} < H < 1$. We prove that the approximation of the derivative of self-intersection local time, defined as

$$\alpha_\varepsilon = \int_0^T \int_0^t p'_\varepsilon(B_t - B_s) \mathrm{d}s \mathrm{d}t,$$

where $p_\varepsilon(x)$ is the heat kernel, satisfies a central limit theorem when renormalized by $\varepsilon^{\frac{3}{2} - \frac{1}{H}}$. We prove as well that for $q \geq 2$, the q-th chaotic component of α_ε converges in L^2 when $\frac{2}{3} < H < \frac{3}{4}$, and satisfies a central limit theorem when renormalized by a multiplicative factor $\varepsilon^{1-\frac{3}{4H}}$ in the case $\frac{3}{4} < H < \frac{4q-3}{4q-2}$.

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